Math 300 **Spring 2009**

Final Exam

May 15, 2009

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Name

Directions: Be sure to include in-line citations of pertinent theorems and propositions. Include any figures used in solving a problem. **Only write on one side of each page.**
"'Know thyself?' If I knew myself, I'd run away." – Johann von Goethe

Do any five (5) of the following.

Extra Credit Make a list of statements that hold in real Euclidean planes but do not hold in real hyperbolic planes. One point (up to 6) for each correct statement that is not in the list of ten in Exercise 1 of Chapter 6 in the textbook. One point off (down to 0) for each incorrect statement.

1. Show that any statement in the language of geometry that is a theorem in Euclidean geometry and whose negation is a theorem in hyperbolic geometry is equivalent to Hilbert's parallel postulate. Hint: Let N denote the axioms of neutral geometry, H Hilbert's parallel postulate and T the theorem in Euclidean geometry that is false in hyperbolic geometry. Then part of what we know is $(N \,\&\, H) \Longrightarrow T$. You are asked to show that given N then
$$T \Longleftrightarrow H.$$

2. Using any result through Meta Mathematical Theorem 1, prove the Corollary to Meta Mathematical Theorem 1.

   If Euclidean geometry is consistent then Hilbert's parallel property is independent of the axioms of neutral geometry.

3. Prove that we can **construct** (using straightedge and compass) a common perpendicular to the two divergently parallel Poincaré lines shown on the board. Specifically, give the center and radius of the appropriate circle and explain why it has the required properties.

4. Let l be a line in the Poincaré Half-plane model of hyperbolic geometry that is a vertical open ray (see the figure on the board). Let P be a point in the half-plane that is not incident with l. Carefully describe both limiting parallel rays to l from P and explain why they satisfy the definition of limiting parallel rays.

5. Prove directly that Incidence Axiom 3 holds in the Poincaré Half-plane interpretation.

6. Let l be a line in the Poincaré Disk model of hyperbolic geometry that is not a diameter of γ_P.Let P be a point interior to γ_P that is not incident with l. Construct one of the limiting parallel rays to l from P and explain why it satisfies the definition of limiting parallel ray. Specifically, describe the center and radius of the appropriate circle δ and explain why that circle satisfies all of the necessary properties.

7. Using any result in neutral geometry, prove that Wallis' Postulate implies Hilbert's Parallel Postulate.

   [Wallis' Postulate is: Given any triangle $\triangle ABC$ and given any segment DE.There exists a triangle $\triangle DEF$ having DE as one of its sides such that $\triangle DEF$ is similar to $\triangle ABC$.]

Figure 1:

Figure 2: